Some solutions for one of the cosmological constant problems

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March 17-18, 2016

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Old Cosmological Problem

If the dark energy is the vacuum energy,
the quantum corrections from the matter diverge $\sim \Lambda_{\rm cutoff}^4$.

$$\rho_{\rm vacuum} = \frac{1}{(2\pi)^3} \int d^3k \frac{1}{2} \sqrt{k^2 + m^2} \sim \Lambda_{\rm cutoff}^4$$

$\Lambda_{\rm cutoff}$: cutoff scale
If the supersymmetry is restored in the high energy,
the vacuum energy by the quantum corrections $\sim \Lambda_{\rm cutoff}^2 \Lambda_{\not{\rm SUSY}}^2$

$$\rho_{\rm vacuum} = \frac{1}{(2\pi)^3} \int d^3k \frac{1}{2} \left(\sqrt{k^2 + m_{\rm boson}^2} - \sqrt{k^2 + m_{\rm fermion}^2} \right)$$
$$\sim \Lambda_{\rm cutoff}^2 \Lambda_{\not{\rm SUSY}}^2$$

$\Lambda_{\not{\rm SUSY}}$: the scale of the supersymmetry breaking.
$\Lambda_{\not{\rm SUSY}}^2 = m_{\rm boson}^2 - m_{\rm fermion}^2$.

If we use the counter term in order to obtain the very small vacuum energy $\left(10^{-3}\,\mathrm{eV}\right)^4$, we need very very fine-tuning and extremely unnatural.

Maybe we do not understand quantum gravity?

We will discuss later.

The above problems could be clues to understand the gravity.

Unimodular gravity

- J. L. Anderson and D. Finkelstein, "Cosmological constant and fundamental length," Am. J. Phys. **39** (1971) 901. doi:10.1119/1.1986321
- W. Buchmuller and N. Dragon, "Einstein Gravity From Restricted Coordinate Invariance," Phys. Lett. B **207** (1988) 292. doi:10.1016/0370-2693(88)90577-1
- M. Henneaux and C. Teitelboim, "The Cosmological Constant and General Covariance," Phys. Lett. B **222** (1989) 195. doi:10.1016/0370-2693(89)91251-3
- W. G. Unruh, "A Unimodular Theory of Canonical Quantum Gravity," Phys. Rev. D **40** (1989) 1048. doi:10.1103/PhysRevD.40.1048
- Y. J. Ng and H. van Dam, "Unimodular Theory of Gravity and the Cosmological Constant," J. Math. Phys. **32** (1991) 1337. doi:10.1063/1.529283

etc.

Unimodular constraint

$$\sqrt{-g} = 1$$

$\Rightarrow g^{\mu\nu}\delta g_{\mu\nu} = 0 \Rightarrow$ Cosmological constant is irrelevant
Lose full invariance under reparametrization (only volume preserving one).

Lagrangian formalism constraint $\Leftarrow$ Lagrange multiplier field λ

$$S = \int d^4x \left\{ \sqrt{-g} \left(\mathcal{L}_{\mathrm{gravity}} - \lambda \right) + \lambda \right\} + S_{\mathrm{matter}}$$

S_{matter}: action of matters,
$\mathcal{L}_{\mathrm{gravity}}$: Lagrangian density of arbitrary gravity models.

Divide the gravity Lagrangian density

$$\mathcal{L}_{\rm gravity} = \mathcal{L}^{(0)}_{\rm gravity} - \Lambda$$

Redefine λ: $\lambda \to \lambda - \Lambda \Rightarrow$

$$S = \int d^4 x \left\{ \sqrt{-g} \left(\mathcal{L}^{(0)}_{\rm gravity} - \lambda \right) + \lambda \right\} + S_{\rm matter} + \Lambda \int d^4 x \, .$$

Last term $\Lambda \int d^4x$ does not depend on any dynamical variable

$\Rightarrow$ may drop the last term.

Obtained action does not include the cosmological constant.
Cosmological constant Λ does not affect the dynamics even in the action.
Cosmological constant may include the large quantum corrections from matters to the vacuum energy.

$\Rightarrow$ Large quantum corrections can be tuned to vanish.

Generalization of Unimodular Gravity

- S. Nojiri, "Some solutions for one of the cosmological constant problems," Mod. Phys. Lett. A **31** (2016) no.37, 1650213, [arXiv:1601.02203 [hep-th]].

$$S = \int d^4 x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda \left(1 - \frac{1}{\mu^4} \nabla_\mu J^\mu \right) \right\} + S_{\rm matter}$$

μ: a constant with a mass dimension,
J^μ: a general vector quantity, ∇_μ: a covariant derivative
$\mathcal{L}_{\rm gravity} = \mathcal{L}^{(0)}_{\rm gravity} - \Lambda$, $\lambda \to \lambda - \Lambda \Rightarrow$

$$S = \int d^4 x \sqrt{-g} \left\{ \mathcal{L}^{(0)}_{\rm gravity} - \lambda \left(1 - \frac{1}{\mu^4} \nabla_\mu J^\mu \right) \right\} + S_{\rm matter} - \frac{\Lambda}{\mu^4} \int d^4 x \sqrt{-g} \nabla_\mu J^\mu$$

The integrand in the last term is total derivative
$\Rightarrow$ the last term does not affect any dynamics and we may drop the last term, again.

We may choose $\nabla_\mu J^\mu$ to be a topological invariant like

- Gauss-Bonnet invariant

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

- For abelian gauge theory,

$$I \equiv \frac{1}{4!}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- Instanton density for non-abelian abelian gauge theory,

$$I \equiv \frac{1}{4!}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\, F_{\mu\nu}F_{\rho\sigma}$$

More Generalization

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda + \mathcal{L}_\lambda \left(\partial_\mu, \partial_\mu \lambda, \varphi_i \right) \right\} + S_{\rm matter} \, ,$$

$\mathcal{L}_\lambda \left(\partial_\mu, \partial_\mu \lambda, \varphi_i \right)$: including the derivatives of λ and other fields φ_i,
not including λ without derivative.

Cosmological constant Λ can be absorbed into the redefinition of λ,
$\lambda \to \lambda - \Lambda$.

Examples:

- Massless scalar field, $\mathcal{L}_\lambda \left(\partial_\mu, \partial_\mu \lambda, \varphi_i \right) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \lambda \partial_\nu \lambda$.
- k-essence $\mathcal{L}_\lambda \left(\partial_\mu, \partial_\mu \lambda, \varphi_i \right) = \mathcal{L} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \lambda \partial_\nu \lambda \right)$.
- Galileon model

These models can describe more realistic and complex evolutions of the universe.

Simplest model

$J^\mu \propto \partial^\mu \varphi$:

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda \left(1 + \frac{1}{\mu^3} \nabla_\mu \partial^\mu \varphi \right) \right\} + S_{\rm matter}$$
$$= \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda + \frac{1}{\mu^3} \partial_\mu \lambda \partial^\mu \varphi \right\} + S_{\rm matter}$$

The term $\frac{1}{\mu^3}\partial_\mu \lambda \partial^\mu \varphi \Rightarrow$ this model may include a ghost.

Redefinitionof scalar fields φ and λ,

$$\varphi = \frac{1}{\sqrt{2}} \left(\eta + \xi \right) , \quad \lambda = \frac{\mu^3}{\sqrt{2}} \left(\eta - \xi \right) ,$$

$\Rightarrow$

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \frac{1}{2}\partial_\mu \xi \partial^\mu \xi + \frac{1}{2}\partial_\mu \eta \partial^\mu \eta + \frac{\mu^3}{\sqrt{2}} \left(\eta - \xi\right) \right\} + S_{\rm matter}$$

η generates the negative norm state and therefore η is a ghost.
$\Rightarrow$ Introducing the fermionic (Grassmann odd) ghosts b and c,

$$S' = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda + \frac{1}{\mu^3}\partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c \right\} + S_{\rm matter} \, .$$

Invariant under BRS transformation

$$\delta\lambda = \delta c = 0 \, , \quad \delta\varphi = \epsilon c \, , \quad \delta b = \frac{1}{\mu^3}\epsilon\lambda$$

ϵ: fermionic parameter.

Defining the physical states as the states invariant under the BRS transformation, the negative norm states can be consistently removed.

- T. Kugo and I. Ojima, "Manifestly Covariant Canonical Formulation of Yang-Mills Field Theories: Physical State Subsidiary Conditions and Physical S Matrix Unitarity," Phys. Lett. B **73** (1978) 459. doi:10.1016/0370-2693(78)90765-7
- T. Kugo and I. Ojima, "Local Covariant Operator Formalism of Nonabelian Gauge Theories and Quark Confinement Problem," Prog. Theor. Phys. Suppl. **66** (1979) 1. doi:10.1143/PTPS.66.1

Conserved ghost number, 1 for c and -1 for b and ϵ

(λ, φ, b, c): a quartet in Kugo-Ojima's quartet mechanism.

Topological Field Theory

Lagrangian density,

$$\mathcal{L} = -\lambda + \frac{1}{\mu^3}\partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c$$

regarded as the Lagrangian density of a topological field theory

- E. Witten, "Topological Quantum Field Theory," Commun. Math. Phys. **117** (1988) 353. doi:10.1007/BF01223371

Lagrangian density is BRS exact, that is, given by the BRS transformation of some quantity.

Start with field theory of φ but $\mathcal{L}_\varphi = 0$
$\Rightarrow$ Under any transformation of φ, the Lagrangian density is trivially invariant
$\Rightarrow$ gauge theory.

Gauge condition,

$$1 + \frac{1}{\mu^3}\nabla_\mu \partial^\mu \varphi = 0$$

Gauge-fixing Lagrangian + ghost Lagrangian = BRS transformation of $-b\left(1 + \frac{1}{\mu^3}\nabla_\mu \partial^\mu \varphi\right)$.

$$\begin{aligned}
&\delta\left(-b\left(1 + \frac{1}{\mu^3}\nabla_\mu \partial^\mu \varphi\right)\right) \\
&= \epsilon\left(-\lambda\left(1 + \frac{1}{\mu^3}\nabla_\mu \partial^\mu \varphi\right) + b\nabla_\mu \partial^\mu c\right) \\
&= \epsilon\left(\mathcal{L} + (\text{total derivative terms})\right)
\end{aligned}$$

- T. Kugo and S. Uehara, Nucl. Phys. B **197** (1982) 378. doi:10.1016/0550-3213(82)90449-7

Lagrangian density: BRS exact up to total derivative.

More about BRS symmetry

Residual gauge symmetry

Gauge condition

$$1 + \frac{1}{\mu^3} \nabla_\mu \partial^\mu \varphi = 0$$

is invariant under the (residual) gauge transformation,

$$\varphi \to \varphi + \delta\varphi \,, \quad \nabla_\mu \partial^\mu \delta\varphi = 0 \,.$$

We can choose (restrict to be) the initial condition where φ is a constant or even zero.

Nakanishi-Lautrup field

λ: Nakanishi-Lautrup field, BRS exact, $\delta b = \frac{1}{\mu^3}\epsilon\lambda$
$\Rightarrow$ Vacuum expectation value of λ should vanish.

If does not vanish, Spontaneous Breakdown of BRS symmetry.
Cannot consistently impose the physical state condition?

Zero-mode (non-oscillating mode) does not generate negative norm states.

Action is also invariant

under (infinite numbers of) deformed BRS transformation,

$$\delta\lambda = \delta c = 0\,, \quad \delta\varphi = \epsilon c\,, \quad \delta b = \frac{1}{\mu^3}\epsilon\left(\lambda + \lambda_0\right)\,.$$

λ_0 satisfies the condition $\nabla_\mu \partial^\mu \lambda_0 = 0$.

If $\lambda_0 = \Lambda$ (cosmological constant) (might be unnatural),

$\Rightarrow \lambda + \Lambda$: BRS exact.

λ exactly cancells the cosmological constant Λ in the physical states.

Further more generalization

Vacuum energy is not only the quantum corrections.
General quantum corrections from the matter,

$$\mathcal{L}_{\rm qc} = \alpha R + \beta R^2 + \gamma R_{\mu\nu} R^{\mu\nu} + \delta R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \, .$$

Coefficient α diverges quadrutically, β, γ, δ logarithmically.
$\Rightarrow$ Further more generation

$$\begin{aligned}
\mathcal{L} =& - \Lambda - \lambda_{(\Lambda)} + \left(\alpha + \lambda_{(\alpha)}\right) R + \left(\beta + \lambda_{(\beta)}\right) R^2 + \left(\gamma + \lambda_{(\gamma)}\right) R_{\mu\nu} R^{\mu\nu} \\
& + \left(\delta + \lambda_{(\delta)}\right) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\
& + \frac{1}{\mu^3} \partial_\mu \lambda_{(\Lambda)} \partial^\mu \varphi_{(\Lambda)} - \partial_\mu b_{(\Lambda)} \partial^\mu c_{(\Lambda)} + \frac{1}{\mu} \partial_\mu \lambda_{(\alpha)} \partial^\mu \varphi_{(\alpha)} - \partial_\mu b_{(\alpha)} \partial^\mu c_{(\alpha)} \\
& + \mu \partial_\mu \lambda_{(\beta)} \partial^\mu \varphi_{(\beta)} - \partial_\mu b_{(\beta)} \partial^\mu c_{(\beta)} + \mu \partial_\mu \lambda_{(\gamma)} \partial^\mu \varphi_{(\gamma)} - \partial_\mu b_{(\gamma)} \partial^\mu c_{(\gamma)} \\
& + \mu \partial_\mu \lambda_{(\delta)} \partial^\mu \varphi_{(\delta)} - \partial_\mu b_{(\delta)} \partial^\mu c_{(\delta)} \, .
\end{aligned}$$

Λ, α, β, γ, and δ may include the divergences due to the quantum corrections from the matters.

The redefinitions of the parameters, $\lambda_{(\Lambda)}$, $\lambda_{(\alpha)}$, $\lambda_{(\beta)}$, $\lambda_{(\gamma)}$, and $\lambda_{(\delta)}$,

$$\lambda_{(\Lambda)} \to \lambda_{(\lambda)} - \Lambda\,, \quad \lambda_{(\alpha)} \to \lambda_{(\alpha)} - \alpha\,, \quad \lambda_{(\beta)} \to \lambda_{(\beta)} - \beta\,,$$
$$\lambda_{(\gamma)} \to \lambda_{(\gamma)} - \gamma\,, \quad \lambda_{(\delta)} \to \lambda_{(\delta)} - \delta\,,$$

$\Rightarrow$

$$\begin{aligned}\mathcal{L} =& -\lambda_{(\Lambda)} + \lambda_{(\alpha)} R + \lambda_{(\beta)} R^2 + \lambda_{(\gamma)} R_{\mu\nu} R^{\mu\nu} + \lambda_{(\delta)} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ & + \frac{1}{\mu^3} \partial_\mu \lambda_{(\Lambda)} \partial^\mu \varphi_{(\Lambda)} - \partial_\mu b_{(\Lambda)} \partial^\mu c_{(\Lambda)} + \frac{1}{\mu} \partial_\mu \lambda_{(\alpha)} \partial^\mu \varphi_{(\alpha)} - \partial_\mu b_{(\alpha)} \partial^\mu c_{(\alpha)} \\ & + \mu \partial_\mu \lambda_{(\beta)} \partial^\mu \varphi_{(\beta)} - \partial_\mu b_{(\beta)} \partial^\mu c_{(\beta)} + \mu \partial_\mu \lambda_{(\gamma)} \partial^\mu \varphi_{(\gamma)} - \partial_\mu b_{(\gamma)} \partial^\mu c_{(\gamma)} \\ & + \mu \partial_\mu \lambda_{(\delta)} \partial^\mu \varphi_{(\delta)} - \partial_\mu b_{(\delta)} \partial^\mu c_{(\delta)}\,.\end{aligned}$$

Divergences can be absorbed into the redefinition of $\lambda_{(i)}$, $(i = \Lambda, \alpha, \beta, \gamma, \delta)$
Divergences might become irrelevant for the dynamics.

Lagrangian density: BRS invariant

$$\delta\lambda_{(i)} = \delta c_{(i)} = 0 \,, \quad \delta\varphi_{(i)} i = \epsilon c \,, \quad \delta b_{(i)} = \frac{1}{\mu^k}\epsilon\lambda_{(i)} \,, \quad (i = \Lambda, \alpha, \beta, \gamma, \delta) \ ,$$

$k = 3$ for $i = \Lambda$, $k = 1$ for $i = \alpha$, and $k = -1$ for $i = \beta, \gamma, \delta$.

Lagrangian density: BRS exact,

$$\delta \left(\sum_{i=0,\alpha,\beta,\gamma,\delta} \left(-b_{(i)} \left(\mathcal{O}_{(i)} + \frac{1}{\mu^k} \nabla_\mu \partial^\mu \varphi_{(i)} \right) \right) \right) \\ = \epsilon \left(\mathcal{L} + \left(\text{total derivative terms}\right) \right) \ .$$

$\mathcal{O}_{(i)} = 1,\ R,\ R^2,\ R_{\mu\nu}R^{\mu\nu},\ R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$.

Quantum corrections from the graviton

$\Rightarrow$ infinite numbers of quantum corrections which diverge.

$\mathcal{O}_{(i)}$: possible gravitational operators $\Rightarrow$ further generalization

$$\mathcal{L} = \sum_i \left(\lambda_{(i)} \mathcal{O}_{(i)} + \frac{1}{\mu^{k_{(i)}}} \partial_\mu \lambda_{(i)} \partial^\mu \varphi_{(i)} - \partial_\mu b_{(i)} \partial^\mu c_{(i)} \right) .$$

All the divergence can be absorbed into the redefinition of λ_i.
The Lagrangian density: BRS invariant and BRS exact.

In order to determine the values of $\lambda_{(i)}$, however, we may need infinite numbers of the initial conditions, which might be physically irrelevant and the predictability of the theory could be lost.

Still understand the quantum gravity?
Might be any clue for the quantum gravity.

Cosmology in Simplest Model

Cosmplogical evolution

$$\mathcal{L}_{\rm gravity} = \frac{R}{2\kappa^2} - \Lambda \qquad \text{(Einstein gravity)}$$

R: scalar curvature, κ: gravitational coupling constant.

FRW metric with flat spacial part,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} \left(dx^i\right)^2 \ ,$$

Assume λ, φ only depend on t. $a(t)$: scale factor.

Eq. for φ

$$0 = 1 + \frac{1}{\mu^3}\left(\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt}\right)$$

Hubble rate $H \equiv \frac{1}{a}\frac{da}{dt}$.

Eq. for λ

$$0 = \frac{d^2\lambda}{dt^2} + 3H\frac{d\lambda}{dt}$$

Neglect contributions from matters.
(t, t) and (i, j) components of Einstein equation,

$$\begin{aligned} \frac{3}{\kappa^2}H^2 =& \Lambda + \lambda - \frac{d\lambda}{dt}\frac{d\varphi}{dt}\,, \\ -\frac{1}{\kappa^2}\left(3H^2 + 2\frac{dH}{dt}\right) =& -\Lambda - \lambda - \frac{d\lambda}{dt}\frac{d\varphi}{dt} \end{aligned}$$

Deleting Λ,

$$\frac{1}{\kappa^2}\frac{dH}{dt} = \frac{d\lambda}{dt}\frac{d\varphi}{dt}\,.$$

A solution for λ: $\lambda = \lambda_0$ (constant) $\Rightarrow H = H_0$ (constant)

$$\Rightarrow \quad \lambda_0 = -\Lambda + \frac{3H_0^2}{\kappa^2}\,, \quad \varphi = -\frac{t}{3H_0}$$

$H = H_0 \Rightarrow$ de Sitter space-time but H_0 does not depend on Λ.
H_0 could be determined by initial condition or something else.
Value of Λ is irrelevant for the cosmology.

Summary

- We have proposed a simple and totally covariant model, which is a topologcal field theory and may solve the problem of the quantum corrections from the matters to the vacuum energy.
- The mechanism is similar to that in the unimodular gravity but the variation of the scalar field λ does not give any constraint on the metric like the unimoduar constraint, but the variation of λ gives the equation for another scalar field φ.
- The problem of the quantum corrections reduce to the classical problem of the initial conditions.